Extending QuAK with Nested Quantitative Automata

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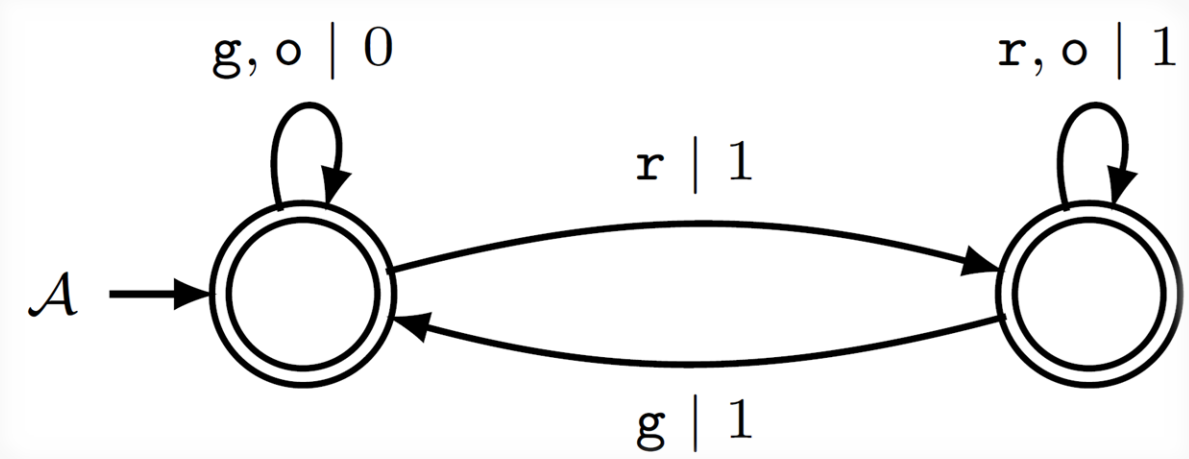
arXiv

GitHub

Quantitative Automata

- Value function $f \in \{\text{Inf}, \text{Sup}, \text{LimInf}, \text{LimSup}, \text{DSum}, \text{LimInfAvg}, \text{LimSupAvg}\}$
- $\mathcal{A}(w) = \sup\{f(\rho) \mid \rho \text{ is an accepting run of } \mathcal{A} \text{ on } w\}$

Example (average utilization rate).



$f = \text{LimSupAvg}$

$w_1 = (o\ r\ g)^\omega$

$\text{LimSupAvg}((0\ 1\ 1)^\omega) = \frac{2}{3}$

$w_2 = o\ r\ g\ o\ o\ r\ g\ o\ o\ o\ o\ r\ g\ \ldots$

$\text{LimSupAvg}(0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ \ldots) = 0$

Chatterjee, Doyen, and Henzinger, "Quantitative Languages", CSL 2008.

Quantitative Automata

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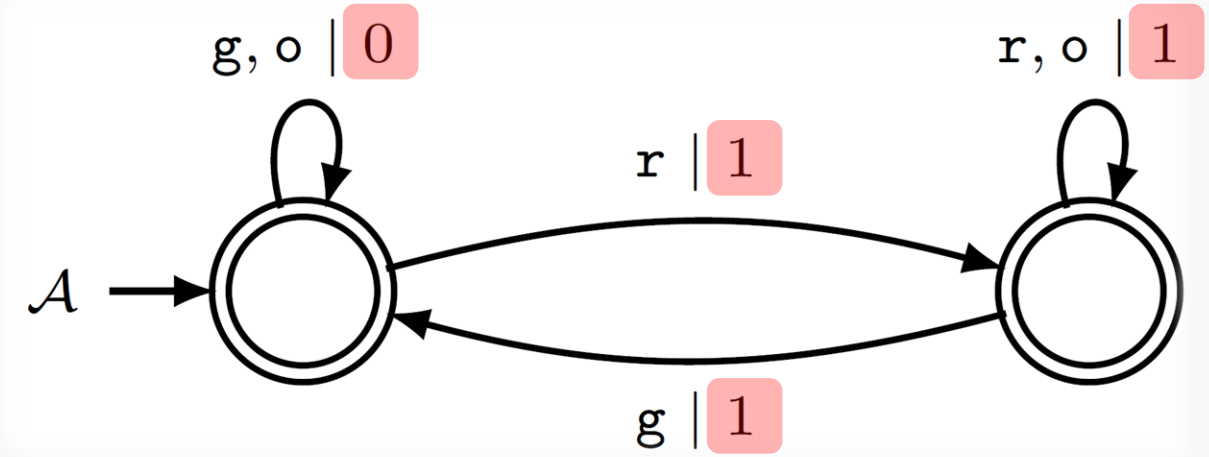


$f = \text{LimSupAvg}$

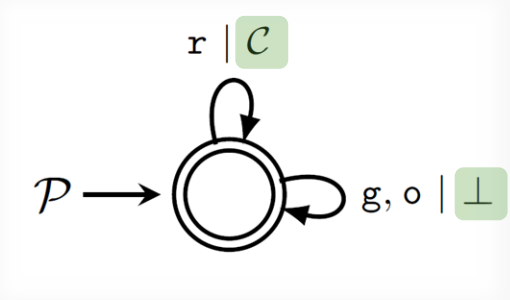
Finitely many transitions with finite weights
↓
Bounded functions

Chatterjee, Doyen, and Henzinger, "Quantitative Languages", CSL 2008.

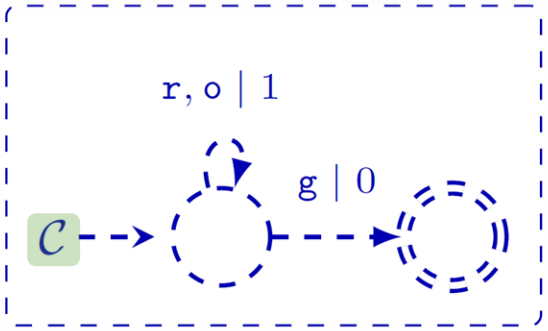
Nested Quantitative Automata

- NQA $\mathbb{A} = (\mathcal{P}, \mathcal{C}_1, \ldots, \mathcal{C}_m)$
- Parent value function $f \in \{\text{Inf}, \text{Sup}, \text{LimInf}, \text{LimSup}, \text{LimInfAvg}, \text{LimSupAvg}\}$
- Child value function $g \in \{\text{Min}, \text{Max}, \text{Sum}^{\text{B}}, \text{Sum}^{+}, \text{Sum}^{-}\}$
- $\mathbb{A}(w) = \sup\{f(\rho) \mid \rho \text{ is an accepting run of } \mathcal{P} \text{ on } w \text{ where all children terminate}\}$

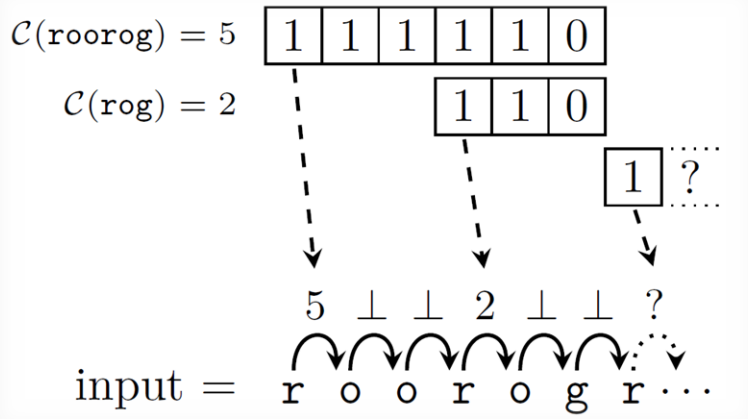
Example (average response time).



$f = \text{LimSupAvg}$



$g = \text{Sum}^{+}$

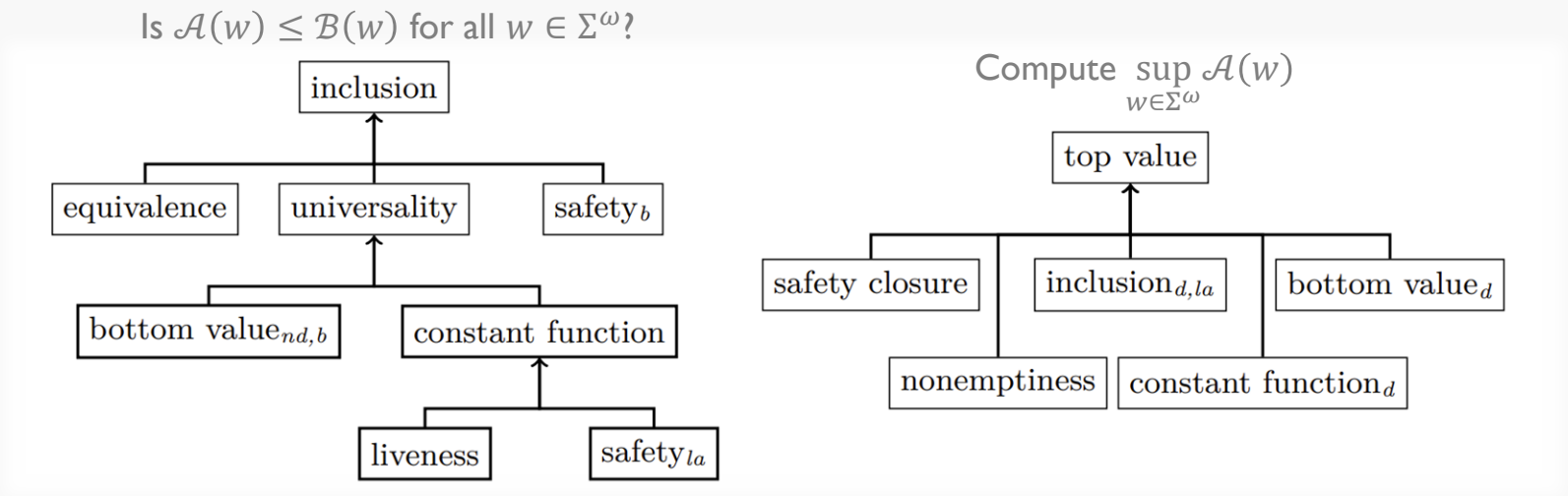


Chatterjee, Henzinger, and Otop, "Nested Weighted Automata", LICS 2015.

QuAK: Quantitative Automata Kit

github.com/ista-vamos/quak

- First tool for quantitative automata
- No dependencies beyond the C++ standard library



Chalupa, Henzinger, Mazzocchi, and Saraç, "QuAK: Quantitative Automata Kit", ISoLA 2024.
Chalupa, Henzinger, Mazzocchi, and Saraç, "Automating the Analysis of Quantitative Automata with QuAK", TACAS 2025.

Contribution

github.com/ista-vamos/nested-quak

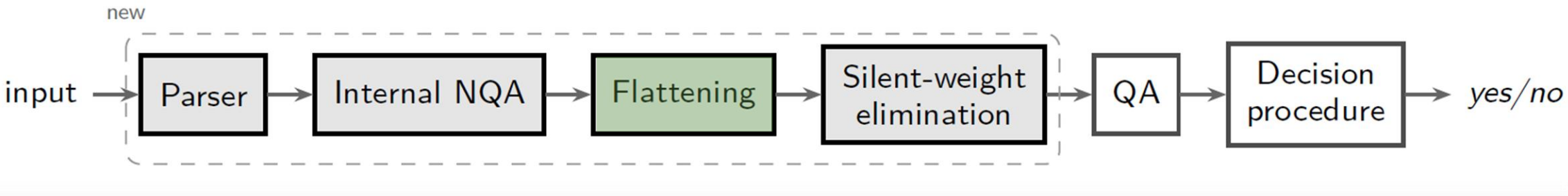
GitHub

First tool support for nested quantitative automata

Table 1: Complexity of decision problems for NQAs with parent aggregator f and child aggregator g [11]. †The (LimInfAvg, Sum^+) case is open; the others are in ExpSpace.

	$f \in$ {Inf, Sup, LimInf, LimSup}		$f \in$ {LimInfAvg, LimSupAvg}	
	Emptiness	Universality	Emptiness	Universality
$g \in \{\text{Min}, \text{Max}, \text{Sum}^B\}$	PSpace	ExpSpace	PSpace	Undecidable
$g \in \{\text{Sum}^+, \text{Sum}^-\}$	PSpace	ExpSpace	ExpSp./Open†	Undecidable

Chatterjee, Henzinger, and Otop, "Nested Weighted Automata", LICS 2015.



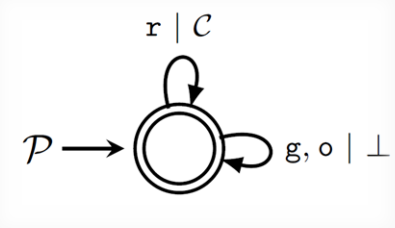
NQA $\mathbb{A} \rightarrow$ QA $\mathcal{A}$ s.t.

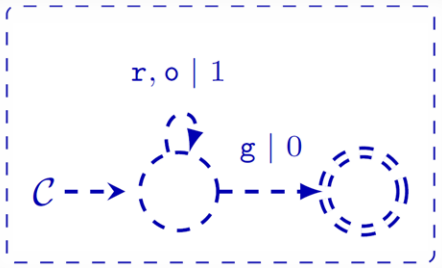
$\mathbb{A}(w) \geq \lambda$ for some (resp. all) $w \in \Sigma^\omega$ iff $\mathcal{A}(w) \geq \lambda'$ for some (resp. all) $w \in \Sigma^\omega$

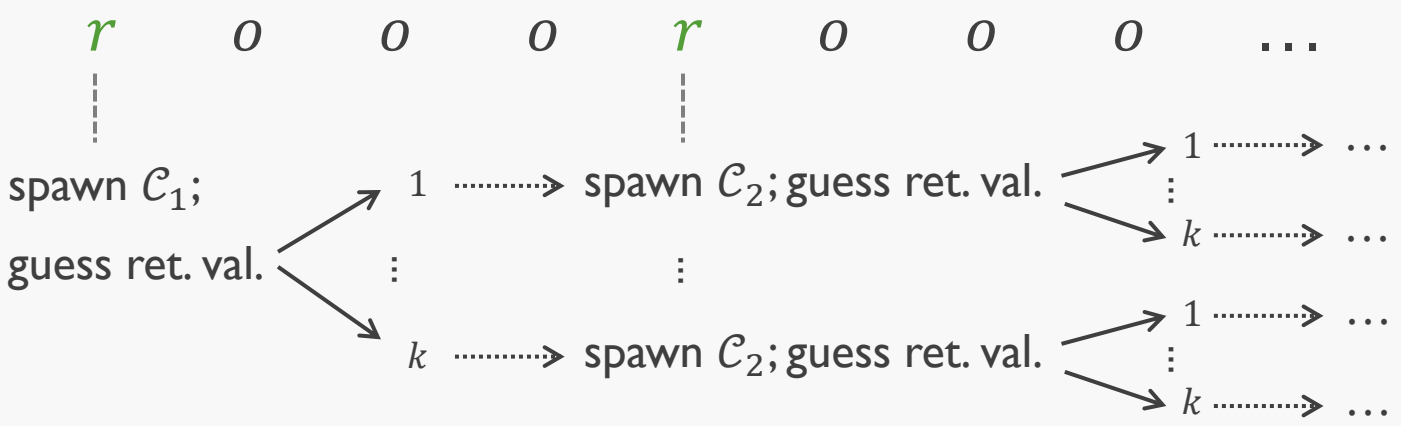
Flattening

Regular flattening

- Exact return value of every active child







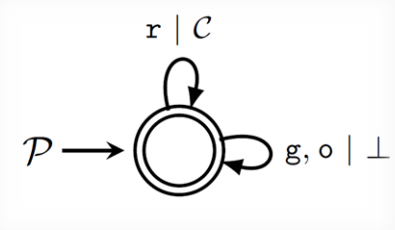
Flattening

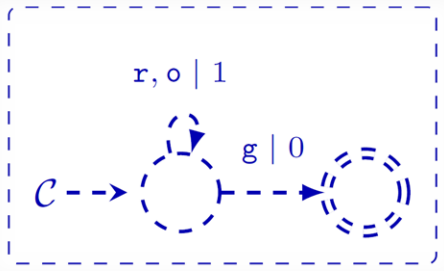
Regular flattening

- Exact return value of every active child

Example shortcut for $(\mathbf{Sup}, \mathbf{Sum}^{+})$ emptiness

- $\exists w : \mathbb{A}(w) \geq \lambda \Leftrightarrow$ there is an accepting run on which some child returns $\geq \lambda$





$r \quad o \quad o \quad o \quad r \quad o \quad o \quad o \quad \ldots$

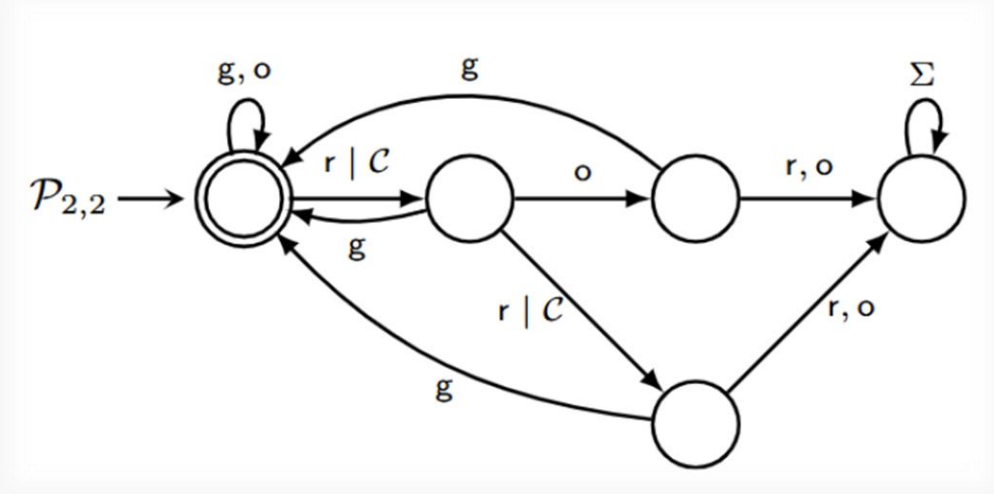
spawn $\mathcal{C}_1$:

- select as witness ⟶ spawn $\mathcal{C}_2$, track it for termination ⟶ …
- do not select ⟶ spawn $\mathcal{C}_2$:
  - select as witness ⟶ …
  - do not select ⟶ …

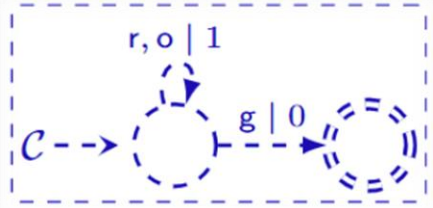
Experiments: Response Time

NQA $\mathbb{A}_{n,k} = (\mathcal{P}_{n,k}, \mathcal{C})$

- At most n requests simultaneously active, each granted within k steps



$f = \mathrm{Sup}$ → maximal response time
$f = \mathrm{LimSupAvg}$ → average response time



$g = \mathrm{Sum}^{+}$

Experiments: Response Time

Is $\mathbb{A}(w) \geq k$ for some $w \in \Sigma^\omega$?

(a) (Sup,Sum$^+$) emptiness

$n \backslash k$	4	8	16	32	64	128	256	512
4	.00	.00	.01	.01	.06	.32	1.5	6.3
8	–	.00	.01	.03	.16	.76	3.3	14
16	–	–	.01	.07	.39	1.7	7.3	32
32	–	–	–	.13	.85	3.8	17	72
64	–	–	–	–	1.3	7.8	35	m
128	–	–	–	–	–	12	71	m
256	–	–	–	–	–	–	m	m
512	–	–	–	–	–	–	–	m

Is $\mathbb{A}(w) \geq k$ for some $w \in \Sigma^\omega$?

(b) (LimSupAvg,Sum$^+$) emptiness

$n \backslash k$	3	4	5	6	7	8	9	10
3	.00	.00	.00	.01	.02	.05	.13	.31
4	–	.00	.00	.01	.07	.28	1.1	3.2
5	–	–	.00	.03	.21	1.3	5.5	20
6	–	–	–	.03	.29	2.8	18	93
7	–	–	–	–	.33	4.3	39	284
8	–	–	–	–	–	4.7	58	m
9	–	–	–	–	–	–	63	m
10	–	–	–	–	–	–	–	m

Is $\mathbb{A}(w) \geq 1$ for all $w \in \Sigma^\omega$?

(c) (Sup,Sum^B) universality

$n \backslash k$	2	3	4	5	6	7	8	9
2	.00	.00	.01	.04	.14	.42	1.2	3.6
3	–	.00	.02	.08	.40	2.3	12	44
4	–	–	.02	.12	1.1	12	159	t
5	–	–	–	.14	2.0	51	t	t
6	–	–	–	–	2.4	136	t	m
7	–	–	–	–	–	177	t	m
8	–	–	–	–	–	–	t	m
9	–	–	–	–	–	–	–	m

m: out of memory (30 GB)
t: out of time (300 seconds)

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512	–	–	–	–	–	–	–	m

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8	–	–	–	–	–	–	t	m
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128	–	–	–	–	–	12	71	m
256	–	–	–	–	–	–	m	m
512	–	–	–	–	–	–	–	m

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5	–	–	–	.14	2.0	51	t	t
6	–	–	–	–	2.4	136	t	m
7	–	–	–	–	–	177	t	m
8	–	–	–	–	–	–	t	m
9	–	–	–	–	–	–	–	m

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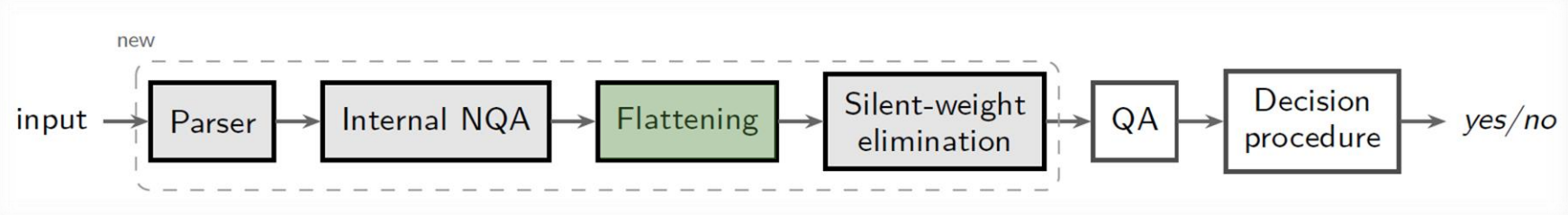
Conclusion

arXiv

GitHub

Contribution

- First tool for analyzing nested quantitative automata



Future Work

- Symbolic algorithms
- Expressive and tractable fragments

Thank you!

C++ library

```cpp
#include "NestedAutomaton.h"

int main() {
    NestedAutomaton* NA = new NestedAutomaton("nested.txt");

    bool nonEmpty = NA->isNonEmpty(LimSupAvg, SumPlus, 5.0);
    bool nonEmpty_SumB = NA->isNonEmpty(LimSup, SumB, 3.0, 10);
    bool universal = NA->isUniversal(Inf, Min_f, 0.0);

    std::size_t numChildren = NA->getChildrenSize();
    ChildAutomaton* child = NA->getChild(1);
    NA->print();

    Automaton* NA_flat = NA->flatten_regular(Max_f);

    bool result = NA_flat->isNonEmpty_withFinal(LimSup, 5.0);

    /* ... */

    delete flat;
    delete NA;
    return 0;
}
```

GitHub

CLI

```
PS C:\Users\egesa\Documents\Repos\nested-quak> ./build/quak-nested
ERROR: No automaton given
Usage: C:\Users\egesa\Documents\Repos\nested-quak\build\quak-nested.exe [-cputime] [-v] [-d] [-debug]
 automaton-file [ACTION ACTION ...]

For non-nested automata, ACTIONs are (VALF = <Inf | Sup | LimInf | LimSup | LimSupAvg | LimInfAvg>):
  non-empty VALF <weight>   (uses Buchi acceptance)
  universal VALF <weight>   (uses Buchi acceptance)

For nested automata (files with @PARENT), ACTIONs are:
  FINVAL = <Max_f | Min_f | SumB | SumPlus | SumMinus>
  non-empty VALF FINVAL <threshold> [bound]
  universal VALF FINVAL <threshold> [bound]  (Inf/Sup/LimInf/LimSup only)
```

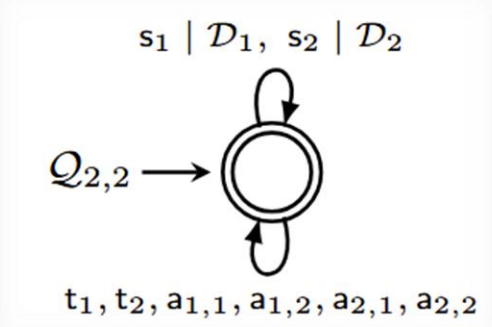
QuAK: Quantitative Automata Kit

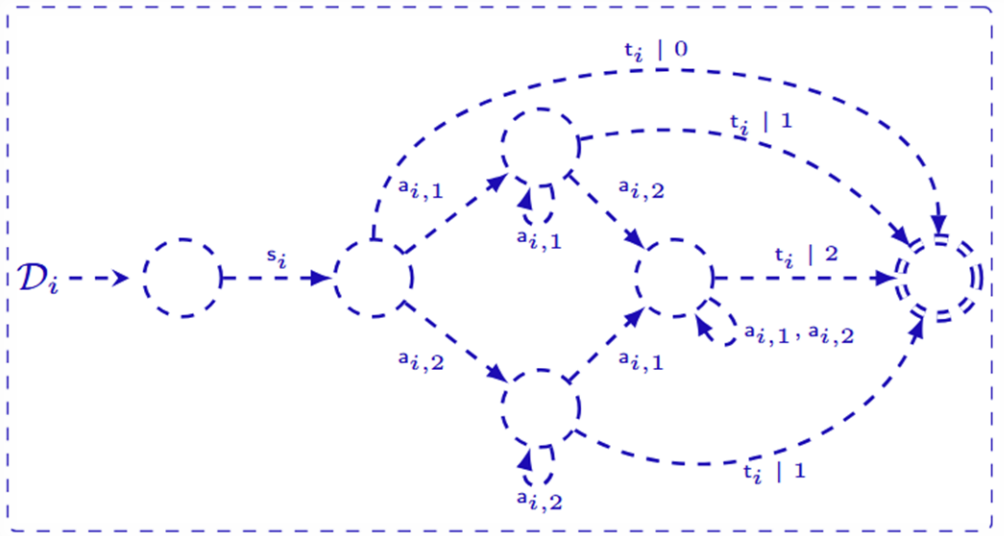
	Inf	Sup, LimInf, LimSup	LimInfAvg, LimSupAvg	DSum
Nonemptiness check	PTIME			
Universality check	PSPACE-complete		Undecidable	Open
Inclusion check	PSPACE-complete		Undecidable	Open
Equivalence check	PSPACE-complete		Undecidable	Open
Top value computation	PTIME			
Safety closure construction	$O(1)$	PTIME		$O(1)$
Safety-liveness decomposition	$O(1)$	PTIME		$O(1)$
Safety check	$O(1)$	PSPACE-complete	EXPSPACE; PSPACE-hard	$O(1)$
Liveness check	PSPACE-complete			
Constant-function check	PSPACE-complete			

Experiments: Resource Consumption

NQA $\mathbb{B}_{n,k} = (\mathcal{Q}_{n,k}, \mathcal{D}_1, \ldots, \mathcal{D}_n)$

- n processes running concurrently, using a set of k resources





$g = \mathrm{Max}$

(self loops with non-i letters are omitted)

(sink state and transitions are omitted)

Experiments: Resource Consumption

Is $\mathbb{A}(w) \geq k$ for some $w \in \Sigma^\omega$?

(a) (Sup,Max) emptiness

$n \backslash k$	1	2	3	4	5	6
1	.00	.00	.00	.00	.00	.00
2	.00	.00	.01	.06	.37	1.9
3	.01	.10	1.1	11	103	m
4	.12	2.4	43	m	m	m
5	1.3	38	m	m	m	m
6	9.9	m	m	m	m	m

Is $\mathbb{A}(w) \geq k$ for some $w \in \Sigma^\omega$?

(b) (LimSupAvg,Max) emptiness

$n \backslash k$	1	2	3	4
1	.00	.00	.00	.00
2	.00	.03	1.1	62
3	.12	48	t	t
4	20	t	t	m

m: out of memory (30 GB)
t: out of time (300 seconds)